

## Instructions

During this session, **no calculator** is permitted.

- Each team has 30 minutes to answer three questions.
- There are three questions in this section, each worth 9 points.
- Write your answers neatly and clearly and label all problems and parts.
- Each team submits **only one** set of answers at the end of the thirty minutes. You must cross out whatever you do not wish to be considered.
- You must show steps and reasoning. Partial credit may be given. All answers in this section should be in exact form, like  $\ln 2$ , unless otherwise indicated.
- Include units where appropriate.
- Make sure the names of all members are written clearly.

School: \_\_\_\_\_

Team Members: \_\_\_\_\_

Score:        /27

1.) A box contains new pencils of standard length, which average about 19 cm long, but after 30 days of heavy use, the average pencil reduces to about 9 cm in length. Assuming the length decreases linearly,

(a) **(2 pts)** Find a function  $L(x)$  that models the length of my pencils after  $x$  days of use. Explain what the slope and vertical intercept of  $L(x)$  mean in the context of this situation.

(b) **(1 pt)** Use the model to find the length of a pencil that has been in use for 15 days.

(c) **(1 pt)** You choose a pencil that is 11 cm long. How many days has it already been used (on average)?

(d) **(1 pt)** Determine the domain of  $L(x)$  in the context of this problem and explain why this domain makes sense in the context of this situation.

(e) **(2 pts)** Do you think  $L(x)$  is perfectly linear? Explain your reasoning.

(f) **(2 pts)** Does  $L(x)$  have an inverse? If it does, find  $L^{-1}(x)$  and explain what it represents in the context of this situation. If it does not, explain why it does not and why it would not make sense in the context of this situation.

2.) Consider the functions  $f(x) = \sin(x)$  and  $g(x) = x^2 + 1$ .

(a) **(3 pts)** Compute  $f(g(x))$  and  $g(f(x))$ . Are these the same function? Explain why or why not.

(b) **(3 pts)** Solve  $f(g(x)) = 0$  and  $g(f(x)) = 1$ .

(c) **(3 pts)** What is the minimum value of  $g(f(x))$ ? For which values of  $x$  is this minimum achieved?

3.) Solve the following logarithm problems.

(a) **(2 pts)** Simplify completely:

$$\log_5(125) - \log_5\left(\frac{1}{5}\right) + \log_5(1).$$

(b) **(3 pts)** Solve for  $x$ :

$$\log_2(x + 5) = \log_2(x - 1) + 2.$$

(c) **(3 pts)** Solve for  $x$ :

$$\log_3(x) + \log_3(x - 6) = 3.$$

Check for extraneous solutions.

(d) **(1 pt)** Without a calculator, determine which is larger:  $\log_2(32)$  or  $\log_3(81)$ . Show your reasoning.