

Calculus Field Day
Activator Challenge

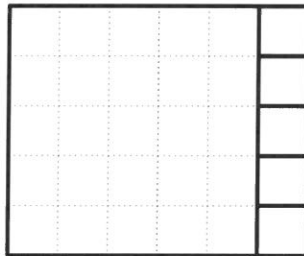
April 16, 2025

Instructions for Activator Challenge:

- The students will select one of the two main problems and if time permits, one of the bonus problems. They work on it for the first 30 minutes.
- During the next 10-minute break, the faculty and the proctors will look through the solutions and come up with an initial ranking and assessment of the winner and runner-up.
- In the next 30 minutes, each team will be invited to make a short (approx. 2 minute) presentation of something notable in their solution. They will be invited in ascending order of ranking, so that weaker teams can have their say before more complete solutions are presented.
- In the next 10 minutes, the proctors/graders conduct a poll to decide which two teams made the best efforts toward a solution.

1. Squaring Rectangles

For this problem, the goal is to dissect a rectangle into the fewest number of squares. For example, a 6×5 rectangle can be divided up into 6 squares as follows:



- ▶ Find a dissection of a 6×5 rectangle using only 5 squares.
- ▶ Find a dissection of a 7×6 rectangle using only 5 squares.
- ▶ Find a dissection of a 13×11 rectangle using only 6 squares.

2. Light Bulbs

You are in a room with 100 light bulbs in a row, each of which can be turned on or off. Initially, all 100 light bulbs are turned off. 100 people visit the room one at a time in a specific order, as follows:

- Person 1 will enter the room and toggle the state (turn on if it's off, turn off if it's on) of every light bulb.
 - Person 2 will enter the room and toggle the state of every 2nd light bulb (so bulbs 2, 4, 6, ..., 10).
 - Person 3 will enter the room and toggle the state of every 3rd light bulb (so bulbs 3, 6, 9, 12, ..., 99).
 - Person 4 will enter the room and toggle every 4th light bulb.
 - This process continues until Person 100 enters the room and toggles the state of every 100th light bulb (just bulb 100).
- At the end of this process, which light bulbs will be on?
Explain your answer.

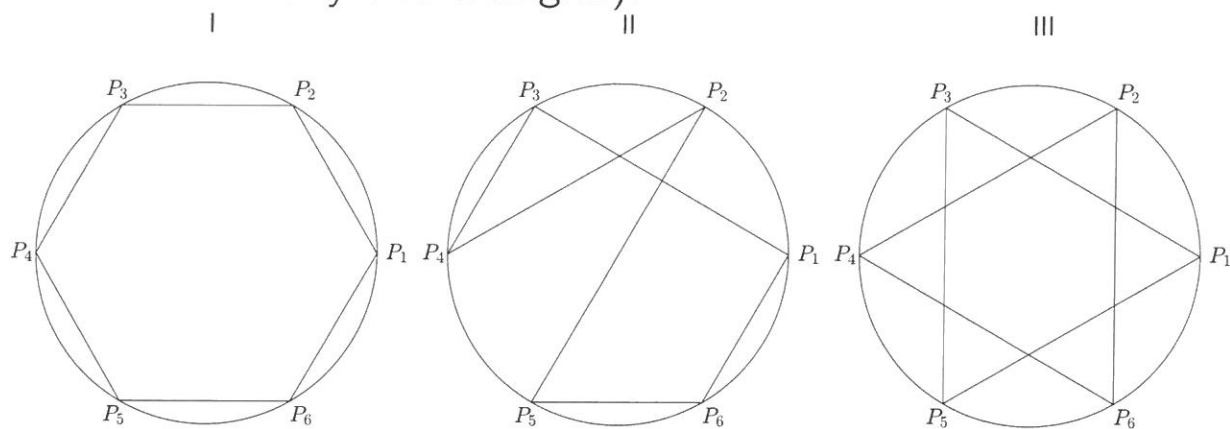
3. Secrets (Bonus)

Each person in a group of 10 friends has a secret that the other people in the group do not know. The only way they can communicate is by sending letters to each other. Each time a person sends a letter to one of their friends, they will share all the secrets they know.

- ▶ What is the fewest number of letters that can be send such each of the 10 friends in the group knows all 10 secrets? Explain why your answer is correct.

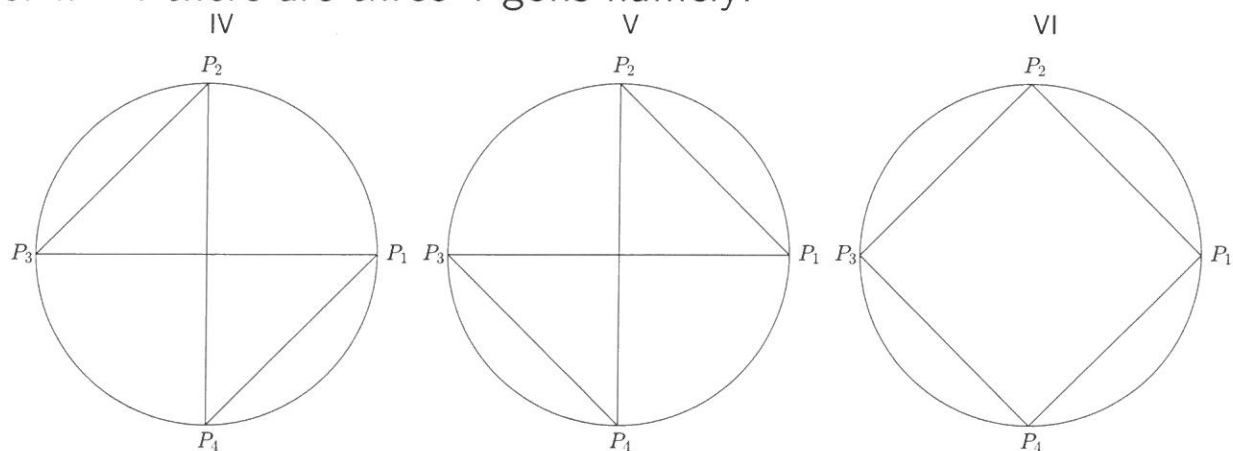
4. Polygons (Bonus)

Suppose we have n points P_1, P_2, \dots, P_n on a circle that are equally spaced out so that they are the vertices of a regular n -gon. We consider n -gons for which P_1, P_2, \dots, P_n are the vertices. For example, in the figure below I and II are 6-gons, but III is not (because it is really two triangles).



4. Polygons (Bonus)

For $n = 4$ there are three 4-gons namely:



- ▶ For $n = 5$, how many 5-gons are there?
- ▶ For arbitrary $n \geq 3$, give a formula for the number of n -gons.
- ▶ Suppose that $n \geq 5$ is a prime number. Two n -gons are equivalent if one can be obtained from the other by rotation. For example, IV and V are equivalent because we get V from IV by rotating over 90° . How many non-equivalent n -gons are there?