AP Calculus BC Instructions - Session 2, No-Calculator Problems

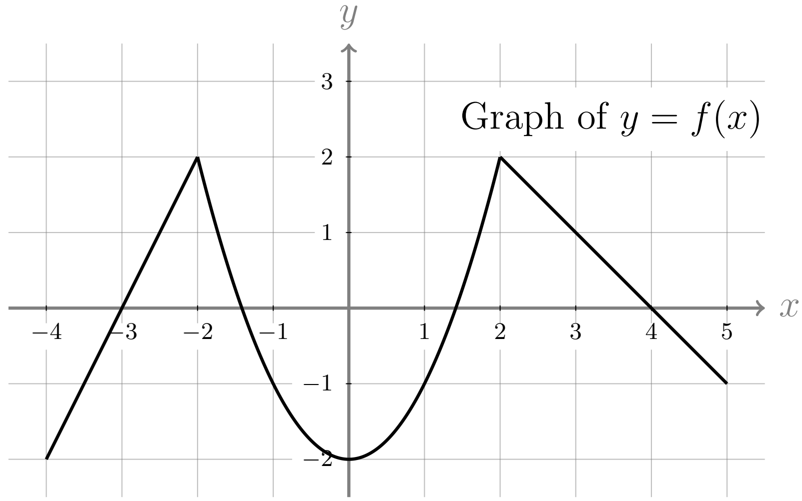
Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the 30 minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 2, use of calculator is not permitted.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as `fnInt(X^2,X,1,5)`
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

4) The function $f(x)$ is continuous for $-4 \le x \le 5$ and its graph consists of two line segments and a portion of the parabolic arc $y = x^2 - 2$ for $-2 < x < 2$, as shown:



Let $F(x) = \int_{-2}^{x} f(x)\,dx$.

(a) Find the value of $F(2)$.

(b) Find the average rate of change of $F(x)$ on the interval $[2, 4]$.

(c) Determine the value(s) of x for which $F(x)$ has a local minimum, and the value(s) of x for which $F(x)$ has a local maximum, for $-4 < x < 5$.

(d) Find all open intervals in $-4 < x < 5$ on which $F(x)$ is concave down.

5) A particle moves in the xy-plane such that its position coordinates $(x(t), y(t))$ at time $t > 0$ are given by

$$x(t) = 2\ln(t) \text{ and } y(t) = t + \frac{1}{t}.$$

(a) Find the velocity vector $v(t)$ and the speed of the particle at time $t > 0$.

(b) Find the total distance traveled by the particle over the time interval $1 \leq t \leq 5$.

(c) Find an equation for the tangent line to the path of the particle at $t = 1$.

(d) Determine the limit as $T \to \infty$ of the particle's average speed on the interval $[1, T]$.

6) Consider the function $f(x)$ defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x+1)^n}{3 \cdot 5^n - 2} = 1 + \frac{(2x+1)}{13} + \frac{(2x+1)^2}{73} + \frac{(2x+1)^3}{373} + \cdots$$

for all real numbers x where the series converges.

(a) Use the limit comparison test with the series $\sum_{n=0}^{\infty} 5^{-n}$ to determine whether the series $f(-1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3 \cdot 5^n - 2}$ converges absolutely.

(b) The sum of the first three terms of the series for $f(-1)$, namely $1 - \frac{1}{13} + \frac{1}{73}$, is used to approximate $f(-1)$. Use the alternating series error bound to determine an upper bound on the error of the approximation.

(c) Determine the radius of convergence of the series for $f(x)$ and the endpoints of the interval of convergence. (You do NOT need to test for convergence at the endpoints.)

(d) Find a series expression for the value of the derivative $f'(0)$.