AP Calculus BC Instructions - Session 1, Calculator Problems

Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the 30 minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 1, use of calculator is permitted.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as `fnInt(X^2,X,1,5)`
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1) The average gas price per gallon $P(t)$ in Boston on March t, 2026 for five values of t is displayed in the table below.

t (days)	1	11	16	21	31
$P(t)$ (dollars)	\$3.09	\$3.53	\$3.67	\$3.79	\$3.99

(a) Using the data in the table, estimate the value of $P'(6)$ and interpret the meaning of $P'(6)$ in the context of the problem.

(b) Use a right-endpoint Riemann sum with the four subintervals indicated by the data in the table to estimate the average gas price during March. Will this estimate be an overestimate or underestimate of the average price? Explain briefly.

Using price and revenue data, an independent watchdog group models the profit earned per gallon of gas by oil producers during the month of March as $f(t) = 0.130 + 0.0129t - 0.0003t^2 + 0.098\sin(t/60)$, for $1 \leq t \leq 31$.

(c) Using $f(t)$, identify the day during March that yielded the most profit per gallon of gas. Justify briefly how you know this is the maximum.

(d) Using $f(t)$, determine the percent increase in the average profit earned on one gallon of gas from the week March 1st to March 8th, to the week March 24th to March 31st.

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2) Let R be the region in the first quadrant bounded above by the graph of $y = \dfrac{4x}{x^2+7}$ and below by the horizontal line $y = \dfrac{1}{2}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = -2$.

(c) The region R is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is a rectangle with height equal to five times the length of the base. Find the volume of the solid.

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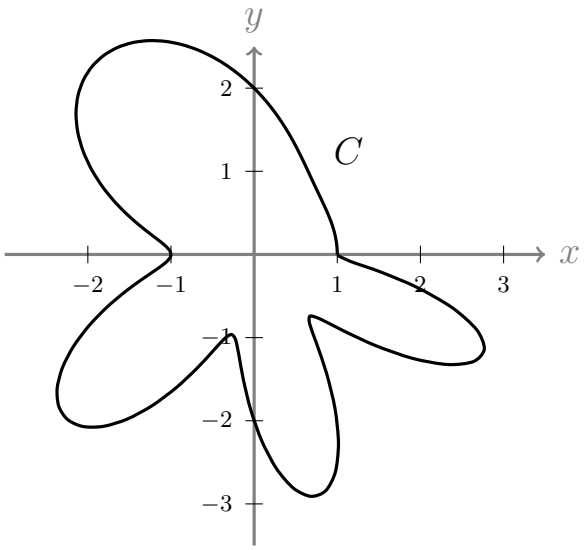
3) Consider the polar curve $C : r = 2 - \cos(2\theta^2/\pi)$ for $0 \leq \theta \leq 2\pi$, as shown:



(a) Find the total area enclosed by C.

(b) Find the total length of the curve C.

(c) Find the slope of the tangent line to C at the point with $0 < \theta < \pi$ where the curve is farthest from the origin.

(d) A particle travels along C so that $d\theta/dt = 4$ for all times t. Find the rate at which the particle's distance from the origin is changing with respect to time when the particle is at the point where $\theta = \pi/2$.
