AP Calculus AB Instructions - Session 1 - Calculator Problems

Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the thirty minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 1, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\, dx$ may not be written as `fnInt(X^2,X,1,5)`
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1.) The rate of sales of a new software product is given by $S(t)$, where S is measured in hundreds of units per month and t is measured in months from the initial release date of January 1, 2012. The software company recorded these sales data:

t (months)	1	2	3	4	5	6	7
$S(t)$ (100s/mo)	1.54	1.88	2.32	3.12	3.78	4.90	6.12

a.) Using a trapezoidal approximation, estimate the number of units the company sold during the second quarter (April 1, 2012, through June 30, 2012).

b.) After looking at these sales figures, a manager suggests that the rate of sales can be modeled by assuming the rate to be initially 120 units/month and to double every 3 months. Write an equation for S based on this model.

c.) Use the model from part (b) to make a prediction for total second quarter sales, then compare this prediction with your estimate from part (a). What is the percentage change?

d.) Use the model to predict the average value of $S(t)$ for the entire first year. Explain what your answer means.

2.) Let R be the region in the first quadrant bounded above by the graph of $y = \dfrac{4x}{x^2 + 7}$ and below by the horizontal line $y = \dfrac{1}{2}$.

a.) Find the area of R.

b.) Find the volume of the solid generated when R is revolved about the line $x = -2$.

c.) The region R is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is a rectangle with height equal to five times the length of the base. Find the volume of the solid.

3.) An architect is designing a grand elliptical gallery known as a **"whispering arch,"** where sound waves travel along the curve of the ceiling and can be heard across the room. The ceiling's cross-sectional profile is modeled by the implicit equation:

$$x\tan(y) + \sin(x+y) = 0$$

where x is the horizontal distance (in meters) from the center of the room and $y > 0$ is the height of the ceiling (in meters) above the floor. After careful analysis, the architect confirms that the point

$$P = (0,\ \pi)$$

lies on the curve and represents the **peak of the arch** at the center of the room.

a.) Find $\frac{dy}{dx}$ in terms of x and y, and determine the slope of the ceiling at P. Interpret your answer in the context of the problem.

b.) The architect needs to know the **concavity** of the arch at its peak to ensure structural integrity. Find $\frac{d^2y}{dx^2}$ at P and determine whether the arch is concave up or concave down there. Justify your answer and explain what this means physically about the shape of the ceiling.

c.) A sound engineer places a speaker at the point on the arch where $x = 0.2$ meters. Using the tangent line at P as a linear approximation, estimate the height y of the ceiling at $x = 0.2$.

d.) Then, the engineer uses a laser sensor and measures the actual ceiling height at $x = 0.2$ to be $y \approx 1.558$ meters. Determine whether the linear approximation from part (c) is an overestimate or underestimate, verify this is consistent with the concavity found in part (b), and compute the **percentage error** of the approximation. Round to three decimal places where needed.