AP Calculus BC Instructions - Session 1, Calculator Problems

Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the 30 minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 1, use of calculator is permitted.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as `fnInt(X^2,X,1,5)`
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1) The stock price for the controversial electric car company Miscar for various days during the month of March are given in the table below.

Date	March 3	March 10	March 18	March 24	March 31
Stock Price	\$300.31	\$252.54	\$228.16	\$258.08	\$249.31

(a) Find the average rate of change of the stock price, with correct units, between March 3 and March 18, and give an interpretation for this value in the context of the problem.

(b) Use a trapezoidal Riemann sum, with intervals as given in the data table, to estimate the average value of the stock price for the 28-day period from March 3 to March 31.

The total value of the stock owned by the CEO of Miscar, L.N. Misque, on the xth day of March, for $1 \leq x \leq 31$, is given by

$$v(x) = 99 - 0.638x + 39.3\cos(x/30) - 21.2\sin(x/10) \text{ billion dollars.}$$

(c) Find the value $v'(10)$, with correct units, and give an interpretation for this value in the context of the problem.

(d) Find the time (to the nearest day in March) at which the total value of the CEO's stock was at its minimum value during March. Briefly explain how you know this is the minimum value.

2) At time t seconds, $0 \leq t \leq 6$, the derivative of the x-coordinate of particle P is given by

$$x'(t) = t^2 - 3t + 2 \text{ meters/sec.}$$

Additionally, the y-coordinate of particle P is given by

$$y(t) = \ln\left(t + \frac{4}{t+1}\right) \text{ meters,}$$

and the position of particle P at time $t = 0$ seconds is $(0, \ln 4)$ meters.

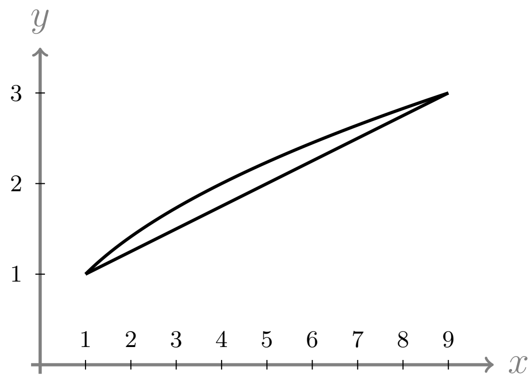
(a) For which t, $0 \leq t \leq 6$, is particle P moving to the right (in the positive x-direction)?

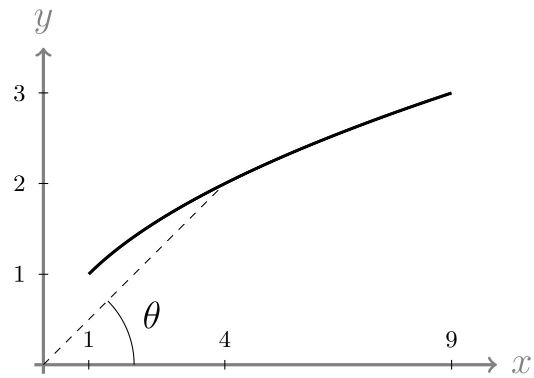
(b) Find the total distance travelled by particle P from time $t = 0$ to time $t = 5$.

(c) Find the time t, $0 \leq t \leq 6$, when the line tangent to the path of the particle is horizontal. Is the particle accelerating to the left or to the right at this time?

(d) Find the position of the particle at time $t = 3$ seconds.

3) Narrow Island, depicted below, is the region lying between the two curves $y = \sqrt{x}$ and $y = (x+3)/4$ between $x = 1$ and $x = 9$.





(a) Find the area of Narrow Island.

(b) A creek runs vertically (parallel to the y-axis) across Narrow Island at its widest point. Find the length of the creek.

At night, a spotlight located at $(0, 0)$ casts a beam of light that traces a path along the curved portion of the coastline $y = \sqrt{x}$. The beam of the spotlight makes an angle θ with the positive x-axis, as shown above.

(c) Find the range of values of θ during which the spotlight is on the coastline.

(d) If the spotlight rotates so that θ increases at a constant rate of one revolution per minute, when the spotlight is at the point $(4, 2)$, how fast is the light traveling along the coastline?

AP Calculus BC Instructions - Session 2, No-Calculator Problems

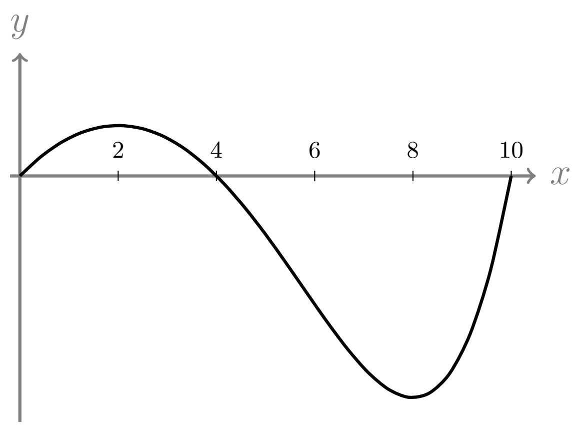
Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the 30 minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 2, use of calculator is not permitted.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as `fnInt(X^2,X,1,5)`
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

4) The graph of a differentiable function $y = f(x)$ for $0 \leq x \leq 10$ is below.



The following information is known about the function $f(x)$:

1. $\int_0^4 f(x)\,dx = 12$.
2. The average value of f on the interval $[4, 10]$ is -12.
3. $\int_0^4 f(x)^2\,dx = 50$.
4. The solutions to $f'(x) = 0$ are $x = 2$, $x = 8$.

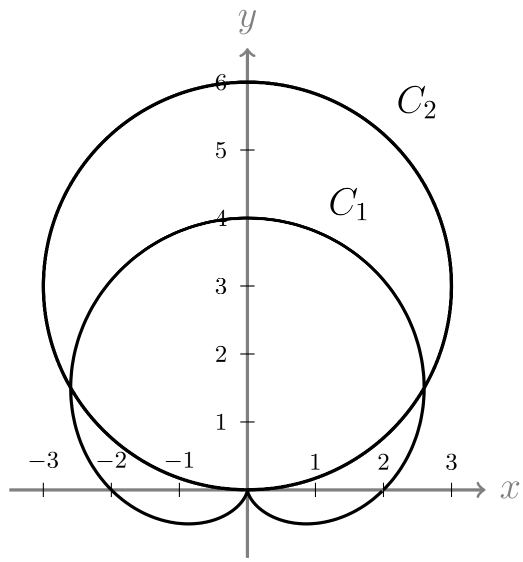
(a) Evaluate $\int_0^{10} f(x)\,dx$. Show the calculations leading to your answer.

(b) For the function $g(x) = \int_2^x f(t)\,dt$, identify all values x with $0 \leq x \leq 10$ for which $g(x)$ is (i) increasing, (ii) decreasing, (iii), concave up, (iv) concave down.

(c) What is the average value of the function $h(x) = f(2x + 2) + 3x^2$ on the interval $-1 \leq x \leq 1$? Show the calculations leading to your answer.

(d) The region under the graph of $y = f(x)$ for $0 \leq x \leq 4$ and above the line $y = 0$ is revolved around the line $y = 0$. Determine the volume of the resulting solid.

5) Consider the polar curves $C_1 : r = 2 + 2\sin\theta$ and $C_2 : r = 6\sin\theta$.



(a) Find the intersection points of curves C_1 and C_2.

(b) Calculate the area of the crescent-shaped region inside C_2 but outside C_1.

(c) Set up, but DO NOT EVALUATE, an integral giving the arclength of curve C_1.

(d) The distance between the curves C_1 and C_2 varies as θ varies. Find the rate at which the distance between the curves is changing when $\theta = \pi/3$.

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6) Consider the function $f(x)$ defined by the following integral:

$$f(x) = \int_0^{x^2} e^{-t^2}\ dt.$$

(a) Explain briefly why $f(x)$ is differentiable, and find the derivative $f'(x)$.

(b) Find all local minima and local maxima of the function $f(x)$.

(c) Using the Maclaurin series for e^{-t^2}, find the Maclaurin series for $f(x)$.

(d) Using the Maclaurin series, give an infinite series expression for the value of $f(1)$. Use the alternating series test to give an upper bound on the error obtained by approximating the value of the series by the sum of its first three non-zero terms.