AP Calculus AB Instructions - Session 2, No Calculator Problems

Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the thirty minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 2, use of calculator is not permitted.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifocations require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_{1}^{5} x^2 \, dx$ may not be written as `fnInt(X^2,X,1,5)`

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

4.) As Goldilocks enters a strange home she notices the thermostat is set to 70° F. Upon seeing some porridge on the table she naturally decides to try it. The porridge is too hot! By her estimate it is just below boiling at 200° F. Luckily, what Goldilocks lacks in manners she makes up for in math skills. She knows that t minutes from now the temperature of the porridge, $P(t)$, satisfies the differential equation $\dfrac{dP}{dt} = \frac{1}{5}(70 - P)$.

     a.) A slope field for the differential equation $\dfrac{dP}{dt} = \frac{1}{5}(70 - P)$ is shown. Sketch the solution curve through the point $(0, 200)$.
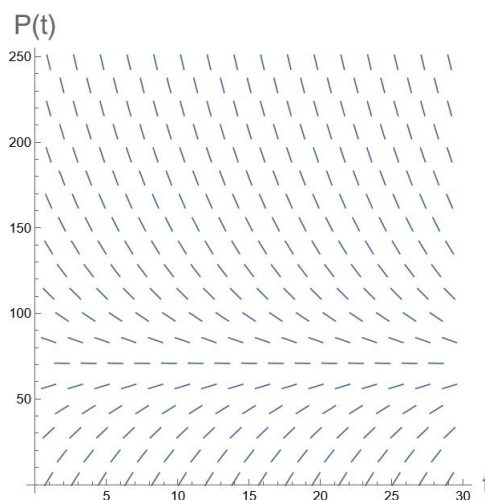
     b.) Goldilocks's patience lasts about 3 minutes. Use the tangent line to the graph of $P(t)$ at $t = 0$ to approximate $P(3)$, the temperature of the porridge after 3 minutes.

     c.) Write an expression for $\dfrac{d^2P}{dt^2}$ in terms of P. Use $\dfrac{d^2P}{dt^2}$ to determine whether Goldilocks's approximation of the temperature of the porridge after 3 minutes (your answer from part (b)) is an overestimate or an underestimate of the temperature. Give a reason for your answer.

5.) Consider the function $f(x) = x(25 - x^2)^{3/2}$ which is graphed below.
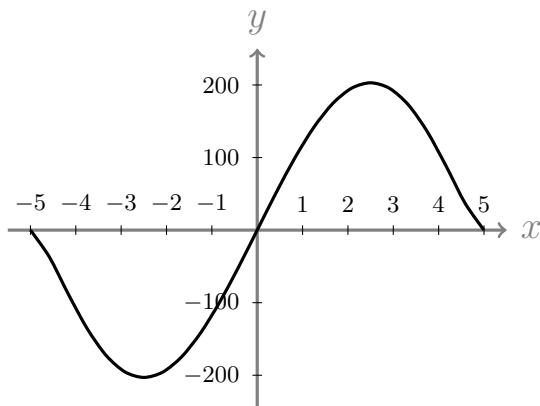
a.) Find an equation for the line tangent to the graph of f at $x = 4$.

b.) Find the area of the region bounded by the graph of f and the positive x-axis.

c.) Set up, but DO NOT EVALUATE, an integral for the volume of the solid obtained by revolving the part of the graph with $0 \leq x \leq 5$ about the x-axis.

6.) Consider the curve given by the equation $x^2y - 16 = y^3$.

a.) Show that $\dfrac{dy}{dx} = \dfrac{2xy}{3y^2 - x^2}$.

b.) Find the coordinates of all points on the curve at which the line tangent to the curve is horizontal, or explain why no such point exists.

c.) Find the coordinates of all points on the curve at which the line tangent to the curve is vertical, or explain why no such point exists.