AP Calculus AB Instructions - Session 1 - Calculator Problems

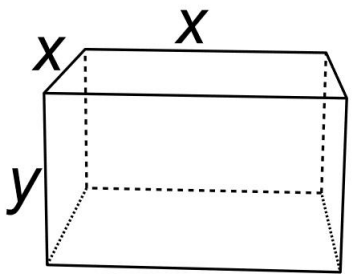
Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the thirty minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 1, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a defnite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as `fnInt(X^2,X,1,5)`
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1.) A rectangular box with square top and bottom is to be made of two materials. The material for the top and bottom costs \$6 per square foot and the material for the sides costs \$3 per square foot.



a.) Suppose the total volume is to be 4 cubic feet. Find the dimensions that minimize the cost.

b.) Suppose the total cost is set to \$36. Find the dimensions that maximize the volume.

2.) A particle moves along the y-axis during a time interval $0 \leq t \leq 4$. The velocity of the particle is given by $v(t) = -6te^{-t^2} + \frac{4}{(t+1)^3} + \frac{t}{2}$, and the particle begins at the origin.

a.) Find an explicit formula for the position of the particle at time t, and then use that expression to find the position of the particle at $t = 3$.

b.) Find the expression for the acceleration of the particle at time t and then use that expression to find the acceleration of the particle at time $t = 2$.

c.) When does the particle change direction? Briefly explain your reasoning.

d.) During which time intervals does the particle slow down? Briefly explain your reasoning.

3.) A chocolatier is filling a life-size mold of a tiger with melted chocolate. The rate of flow of chocolate is modeled by a differentiable function C, where $C(t)$ is measured in cups per minute and t is measured in minutes since pouring began. Selected values of $C(t)$ are given in the table.

t (minutes)	0	8	15	25	30
$C(t)$ (cups per minute)	0	10	17	12	2

a.) Using correct units, interpret the meaning of $\int_8^{30} C(t)\,dt$ in the context of the problem. Use a left Riemann sum with the three subintervals [8,15], [15,25], and [25,30] to approximate the value of $\int_8^{30} C(t)\,dt$.

b.) Approximate $C'(5)$ using the average rate of change of C over the interval $0 \le t \le 8$. Show the work that leads to your answer and include units in your answer.

c.) Must there exist a value of c, for $15 \le c \le 30$, such that $C'(t) = -1$? Justify your answer.

d.) The rate of flow of chocolate, in cups per minute, can also be modeled by $m(t) = 9.743\sin(0.194t - 1.104) + 8.232$ for $0 \le t \le 30$. Using this model, find the average rate of flow of chocolate over the time interval $0 \le t \le 30$. Show the setup for your calculations.