1. The height of a tide t hours after we start observing is given by $H(t) = 5 + A\sin\left(\frac{\pi t}{12.5}\right)$, where H is in feet.

   a) (3 pts) If the maximum height of the tide is 23 feet, what is A?

   b) (6 pts) If we stay for 24 hours, how often do we see this maximum height and when do they occur?

Answers: (a) The largest sine can be is 1 so the max is 5 + A = 23, so A=18, set up 2 pts answer 1 pt

(b) The max occurs when π · t /12.5 = π/ 2 + 2kπ; π · t/ 12.5 = π /2 gives t=6.25, π · t /12.5 = π/ 2 + 2π gives t=31.25 so we only see the max once after 6.25 hrs (reasoning and set up 4 pts answer 2pts?)

2. a) (5 pts) Solve this system of equations algebraically. Show all steps.

$$\begin{cases} 5x + 6y = 7 \\ -x - 4y = 0 \end{cases}$$

   b) (4 pts) Sketch a graph of the system in part a) labelling the scale and the point of intersection.

Answers: 3 pts set up and algebra; 2pts answer – coordinates? Necessary or not for y?

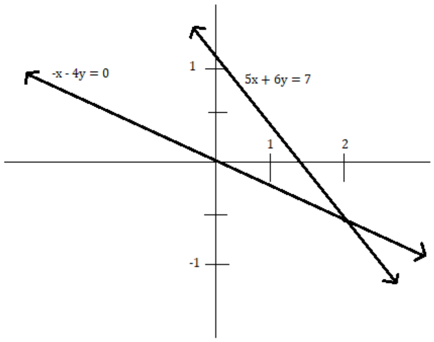
$5x+6y=75(-x-4y=0) \rightarrow 5x+6y=7-5x-20y=0$

$\rightarrow -14y=7 \rightarrow y= -0.5$ substituting into equation: $-x-4y=0$

and solving for x gives: $-x-4(-0.5)=0 \rightarrow x=2$

Intersection point is (2, -0.5)

Graph below: (2pts – one for scale and lines one for intersection and scale)



3. Consider the functions $f(x) = \sqrt{\ln(x)}$ and $g(x) = e^{x^2+1}$.

a) (3 pts) Solve f(g(x)) = 2 and g(f(x)) = e^2

b) (3 pts) For which values of *a* does the equation f(g(x)) = *a* have exactly one solution?

c) (3 pts) Do the graphs of f(g(x)) and $\frac{g(f(x))}{e}$ intersect? Explain your answer.

Solution and grading scheme: We first compute f(g(x)) = sqt (x ^2 + 1) and g(f(x)) = ex.
(a) Solving Sqt(x ^2 + 1) = 2, we get x^ 2 + 1 = 4, so x ^2 = 3 and therefore x = ± sqt(3). Solving ex = e^2 , we get x = e. (1 pt for each composition formulas, and 2 pt for solutions – 1 pt each)

(b) The equation sqt(x^ 2 + 1) = a will have a solution for x only if a ≥ 0. In that case x^2 + 1 = a^2 , and so x^2 = a^2 − 1. This will have a unique solution for x when a = ±1. (2 pt for simplification, 1 pt for conclusion)

(c) If the two graphs intersect, we must have f(g(x)) = g(f(x)) e , i.e. sqt(x^2 + 1) = x, which simplifies to x^2 + 1 = x^2 , or 1 = 0. Therefore, this equation has no solution and so the graphs don't intersect. (2 pts for computation, 1 pt for conclusion)