1. The Johnstown Inclined Plane in Pennsylvania starts on a flat road at a certain elevation above sea level and is one of the longest and steepest hoists in the world. The railway cars travel a distance of 896.5 feet along the inclined plane at an angle of approximately 35.4°, rising to a total height of 1693.5 feet above sea level.



- a) (2 pts) Find the vertical rise of the inclined plane.
- b) (3 pts) Find the elevation above sea level of the flat road (beginning of the inclined plane).
- c) (4 pts) The cars move up the plane at a rate of 300 feet per minute. Find the rate at which they rise vertically from the bottom of the plane.

a. sin(35.4°)= oppositehypotenuse= vertical rise896.5 feet=521.87 feet=vertical rise (2pts)

b. Elevation of lower end of the inclined plane is 1693.5 *feet*-521.87 *feet*=1171.87 *feet* (3 pts)

c. Velocity= distance time  $\rightarrow$  time= distancevelocity= 896.5 ft300 ft/min=2.988 min to reach the top of incline. Using this time and the vertical height and the velocity equation, we can find the rate of vertical rise: Velocity= distance time = 521.87 ft2.988 min=174.64 ft/min. (4 pts - 2pts for time 2pts for ans.) 2. The population of China in 1990 was 1135 (in millions) and was growing by 15 (million) per year. The population of India in 1990 was 873 (in millions) and was growing by 2% per year.

a) (4 pts) Give equations for the population (in millions) of each country where t is the number of years since 1990.

b) (2 pts) Give the population of each of the countries in the year 2023.

c) (3 pts) When will the population of India be equal to the population of China? Give your answer to three decimal places.

Answers: a) C(t) = 1135 + 15t, I(t) = 873(1.02)^t, (1pt initial value 1 pt growth rate for proper model) (b) C(33) = 1630 million, I(33) = 1678.11 million, (1pt each)

(c) t=30.225 (years after 1990) (show some reasoning – 1 pt answer 2 pts)

3. Two plant colonies on Mars have total mass at time t weeks given by  $C(t) = 2t^2 - 3t + 20$  and  $D(t) = t^2 - t + 10$  kilograms.

- a) (3 pts) At what time are the colonies closest in mass?
- b) (4 pts) What is the minimum mass of each colony?
- c) (2 pts) Which population reaches its minimum mass first?

Solution and grading scheme: (a) We need to find the time t when  $C(t) - D(t) = t^2 - 2t + 10$  is smallest. This minimum is achieved at t = 1 week and is equal to 9 kilograms. 2 pt for the setup of the difference (or other equivalent method), 1 pt for finding t = 1

(b) The minimum of C(t) is achieved at t = 3/4 and is equal to 151/8. The minimum of D(t) is achieved at t = 1/2 and is equal to 39/4. 2 pts for each min mass

(c) Since the population governed by C(t) reaches its minimum mass at t = 3/4 and that governed by D(t) achieves its minimum mass at t = 1/2, it follows that the latter reaches its minimum mass first. 2 pt for right conclusion with reasoning.