AP Calculus AB Instructions - Session 2, No Calculator Problems

Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the thirty minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 2, use of calculator is not permitted.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifcations require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, defnitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 \, dx$ may not be written as `fnInt(X^2,X,1,5)`

- Unless otherwise specifed, answers (numeric or algebraic) need not be simplifed. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specifed, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1.) Suppose $g(x)$ is a continuously differentiable function with $g(1) = 2$ and $g'(1) = 3$. Let $h(x) = (g(x))^2$.

    a.) Find $h'(1)$.

    b.) Find the equation of the tangent line to the graph $y = h(x)$ at $x = 1$.

    c.) Evaluate the limit
$$\lim_{x \to 1} \frac{\sqrt{h(x)} - 2}{g(x) - 2}.$$

2.) Let $F(x) = \displaystyle\int_1^x f(t)\ dt$, where $f(t) = \displaystyle\int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\ du$.

    a.) Over the interval $1 \le x \le 10$ is $F(x)$ concave up or down? Explain your reasoning.

    b.) Find $F''(2)$.

    c.) Find $\displaystyle\lim_{x \to 1} \frac{F(x)}{x^2 - 2x + 1}$.

3.) A rectangular box with square top and bottom is to be made of two materials. The material for the top and bottom costs $5 per square foot and the material for the sides costs $3 per square foot.

    a.) Suppose the total volume is to be 45 cubic feet. Find the dimensions that minimize the cost.

    b.) Suppose the total cost is set to $120. Find the dimensions that maximize the volume.