AP Calculus AB Instructions - Session 1 - Calculator Problems

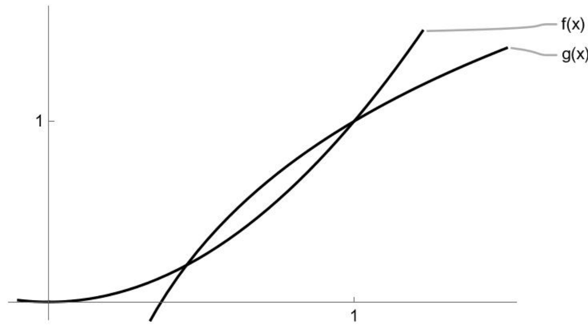
Manage your time carefully. Each team has 30 minutes to answer three questions. Each team submits one set of answers at the end of the thirty minutes.

Cross out any errors you make; erased or crossed-out work will not be scored.

During Session 1, you are permitted to use your calculator to solve an equation, fnd the derivative of a function at a point, or calculate the value of a defnite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifcations require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, defnitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as `fnInt(X^2,X,1,5)`
- Unless otherwise specifed, answers (numeric or algebraic) need not be simplifed. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specifed, your fnal answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1.) Consider the region R between the curves $f(x) = x^2$ and $g(x) = 1 + \ln x$ over the interval $[a, 1]$ where $0 < a < 1$ and $f(a) = g(a)$.



a.) Find the area of the region enclosed by the graphs of f and g.

b.) Find the volume of the washer generated when R is revolved around the x-axis.

c.) Find the volume of the washer generated when R is revolved around the y-axis.

2.) Harriet the snail is moving along a straight, skinny rail. Her velocity at time t is $v_H(t) = \sqrt[3]{\cos(t)}$ for $0 \le t \le 3$. At time $t = 0$ Harriet is at position 2 along the rail.

Thomas the snail is moving along the same straight, skinny rail. His velocity at time t is $v_T(t) = (t-1)(0.8^t)$ for $0 \le t \le 3$. At time $t = 0$ Thomas is at position 3 along the rail.

a.) Find Harriet's and Thomas's positions along the rail at time $t = 2$.

b.) Are Harriet and Thomas racing towards each other or away from each other at time $t = 2$? Explain your reasoning.

c.) Find the total distance travelled by Harriet over the time interval $0 \le t \le 3$.

t (hours)	1	3	6	8	9
$m(t)$ (square feet)	151.02	72.97	18.97	7.19	4.39

3.) At a daycare facility the amount of floor space which is still uncluttered with toys t hours after the facility opens for the day is given by a differentiable function m, where $m(t)$ is measured in square feet. Selected values of $m(t)$ are given in the table above.

a.) Use the data in the table to estimate $m'(7)$. Using correct units, interpret the meaning of $m'(7)$ in the context of the problem.

b.) Explain why there must be at least one time t, for $1 < t < 9$, such that $m'(t) = -18$.

c.) The clear space on the floor, in square feet, can also be modeled by a function C, given by $C(x) = \frac{1}{1000}(-1.3x^4 + 83x^3 - 1476x^2 - 870x + 158005)$, where x is the number of children at the facility. When there are 21 square feet of clear floor space the number of children is increasing at a rate of 0.2 children per hour. According to this model, what is the rate of change of amount of floor space with respect to time, in square feet per hour, at the time when there are 21 square feet of clear floor space?