

2022 Calculus Field Day at Northeastern

April 22, 2022

Calculus BC – Session 2

Directions:

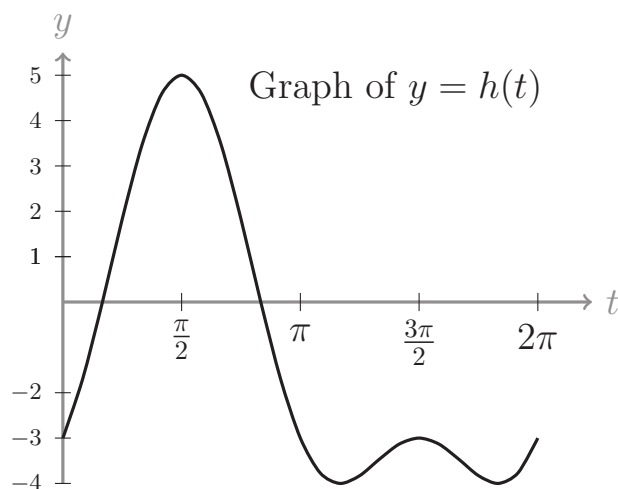
- This is Calculus BC, Session 2. Graphing calculators are NOT ALLOWED for this session.
 - Write your responses as neatly and as clearly as possible. Label all problems and parts.
 - Show all relevant work. A correct answer with missing justification may not receive full points.
 - Numerical answers should be exact or rounded to three decimal places, unless otherwise instructed.
 - There are 3 problems in this session, each worth 10 points in total.
 - Time: 30 minutes.
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Do NOT go to the next page until you are directed to begin.

- 4) A rowboat is tied to a dock, and waves hitting the boat cause it to move up and down slowly. The height in inches of the boat above the dock after t minutes is given by the function

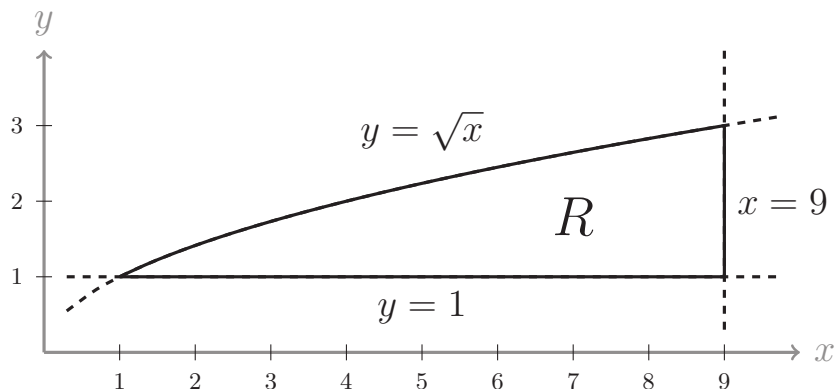
$$h(t) = 4\sin^2(t) + 4\sin(t) - 3$$

whose graph is shown below.



- (a) Calculate the value $h'(\pi)$, including correct units, and briefly describe the meaning of this quantity in the context of the problem.
 - (b) What is the average height of the boat during the interval $0 \leq t \leq 2\pi$ minutes?
 - (c) Between time $t = 0$ minutes and time $t = 2\pi$ minutes, for how much total time is the boat above the surface of the dock?
 - (d) Give an integral expression for the average height of the boat during the time it is above the dock. (You do not need to evaluate the integral.)
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- 5) Let $g(x) = \sqrt{x}$ for $x \geq 1$ and let R be the region lying below the graph of $y = g(x)$, above the line $y = 1$, and to the left of the line $x = 9$:



- (a) Find the area of the region R .
- (b) If the region R is revolved around the x -axis, what is the volume of the resulting solid?
- (c) The region R is the base of a solid whose cross-sections perpendicular to the x -axis are squares. What is the volume of the solid?
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6) A function $f(x)$ is defined by the Maclaurin series

$$f(x) = \frac{1}{4}x - \frac{2}{4^2}x^2 + \frac{3}{4^3}x^3 - \frac{4}{4^4}x^4 + \cdots + (-1)^{n+1} \frac{n}{4^n}x^n + \cdots$$

for all real numbers x such that the series converges.

(a) Find, with justification, the interval of convergence of the Maclaurin series for $f(x)$.

(b) Find the value of the limit $\lim_{x \rightarrow 0} \frac{f(x) - x/4}{x^2}$.

(c) Express the value of $\int_0^1 f(x) dx$ as an infinite series.

(d) Use the infinite series in (c) to give a rational number expression approximating $\int_0^1 f(x) dx$ to within an error of at most $1/1000$, making sure to justify why the error is at most $1/1000$. (You do not need to simplify the rational number expression.)
