2022 Calculus Field Day at Northeastern

April 22, 2022

Calculus BC – Session 1

Directions:

- This is Calculus BC, Session 1. A graphing calculator is REQUIRED for this session.
- Write your responses as neatly and as clearly as possible. Label all problems and parts.
- Show all relevant work. A correct answer with missing justification may not receive full points.
- Numerical answers should be exact or rounded to three decimal places, unless otherwise instructed.
- There are 3 problems in this session, each worth 10 points in total.
- Time: 30 minutes.

Do NOT go to the next page until you are directed to begin.

1) On weekdays, if t is measured in hours since 5am, then the rate of riders getting on the Red Line from 5am to 10am can be modeled by

$$H(t) = 7t^3 - 145t^2 + 772t \text{ people per hour}$$

and the rate of riders leaving the Red Line from 5am to 10am can be modeled by

$$L(t) = 7t^3 - 155.5t^2 + 922.25t - 423.125 \text{ people per hour.}$$

(a) According to the model, between 6am and 9am, how many riders in total enter the Red Line? Give your answer to the nearest whole number.

(b) According to the model, between 6am and 9am, what is the average (mean) number of riders per hour who leave the Red Line? Give your answer to the nearest whole number.

(c) Assuming no one was on the Red Line at 5am, according to the model, how many riders are currently on the Red Line at 7am?

(d) Is the rate of change in the number of riders on the Red Line increasing or decreasing at 8am? Explain your reasoning.

2) A particle travels through the plane so that its velocity vector at time t is $(2\sin(7t+\pi/3), \sqrt{t^3+4})$. At time $t = 0$ seconds, the particle is located at the point $(1, 2)$.

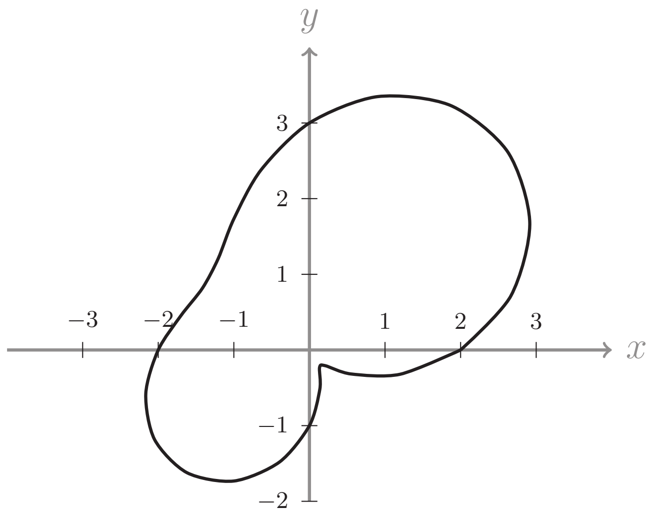
(a) Find an equation for the tangent line to the particle's path at time $t = 0$.

(b) Find the particle's acceleration vector at time $t = 0$.

(c) Find the particle's position at time $t = 2$.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 2$ seconds.

3) Consider the polar curve $r = 2 + \sin(\theta) + \sin(2\theta)$ for $0 \leq \theta \leq 2\pi$:



(a) Find the total area enclosed by the curve.

(b) Find the total arclength of the curve.

(c) Find the value of θ corresponding to the point on the curve whose distance to the origin is maximized.

(d) Find the value of θ corresponding to the point on the curve in the third quadrant where the tangent line is vertical.