SLAC today

Twist and Shout!

To understand physics at the CERN Large Hadron Collider (LHC), we will need to work much harder to compute cross sections for quark and gluon scattering. Gluon-gluon scattering reactions with momentum transfers above 100 GeV are expected to occur at the LHC at the rate of thousands per second. Even reactions with four or five recognizable jets of particles originating from quarks or gluons will be produced at such a high rate that we cannot record every event to permanent storage. The ease with which quarks and gluons can be radiated makes it difficult not only to study the Standard Model but also to search for new physics. We might hope to find the supersymmetric partners of quarks and gluons, or other types of new exotic particles, at the LHC. But the expected rate for producing supersymmetric particles is not so different from the rate for producing a W or Z boson plus four high-energy gluons, or a top quark pair plus two high-energy gluons. We had better understand these complex Standard Model processes well if we wish to model them successfully, and we will need those models to claim any discovery of new physics.

Quarks and gluons interact weakly at high energy, and we can compute their interactions using the Feynman diagrams of quantum chromodynamics (QCD). The scattering of two gluons to two gluons is a textbook-level calculation with four Feynman diagrams; still, it makes a long homework set. As the number of gluons increases, the number of diagrams increases exponentially. The calculation of two gluon scattering to six gluons needs thousands of Feynman diagrams, each one of which would take a page of algebra to write out.

Fortunately, beginning in the 1980's, theorists have devised simpler methods for these computations. At Fermilab, Stephen Parke and Tomasz Taylor made very tricky use of supersymmetry to analyze quark-gluon reactions. They learned a remarkable fact, that certain particular quark-gluon amplitudes, characterized by the polarizations or spins of the incoming and outgoing particles, take a very simple form. For example, the amplitude for two gluons with right-handed circular polarization to scatter to a final state containing only similar right-handed gluons is of this class. After we sum the full set of Feynman diagrams, these amplitudes turn out to be given by a single ratio of products of two-gluon dijet masses. The formula applies no matter how many gluons, with the correct polarizations, are produced.

In 2003, this formula caught the eye of Edward Witten at the Institute for Advanced Study. Witten had dreamed for a long time that problems in QCD could be solved exactly using the formalism of "twistors" invented by Roger Penrose of Oxford University. Twistors replace Lorentz vectors by spin one-half objects, spinors, and then manipulate the components of these spinors as abstract complex numbers. With this change of variables, we can try to use all of the power of the theory of functions of a complex variable to transform the equations and eventually solve them. At intermediate steps in the computation, we find ourselves working with Lorentz vectors with complex components, objects that have no apparent physical meaning. But this sort of problem does not trouble theorists. If at the end of the analysis we obtain a physically sensible result, that is enough.

Witten noticed that the Parke-Taylor formula, written in spinor variables, is an analytic function with simple singularities. This gave clues for generalizations of this formula to include other possible combinations of quark and gluon polarizations. Many QCD experts, including SLAC's Lance Dixon, joined the search. Finally, three postdoctoral fellows at the Institute for Advanced Study, Ruth Britto, Freddy Cachazo, and Bo Feng (BCF), codified the new results in terms of a simple formula. To compute a QCD amplitude, they found, one should cut it into simpler components, shift the momenta of those component amplitudes, and sum these contributions. The shifts they called for preserve the condition that quarks and gluons remain massless, but move the momenta to unphysical complex values. The procedure can be applied recursively until we reduce the original complicated amplitude to simple components.

The BCF procedure turns out to apply not only to QCD amplitudes but also to new physics processes that we will be interested in at the LHC. Many new particles are produced in gluon-gluon reactions. At the LHC, the colliding gluons can radiate additional jets, and this radiation must be understood to define an optimal search strategy. For example, we would like to model the radiation of additional jets in the reaction that produces the Higgs boson in a gluon-gluon collision. With Nigel Glover and Valentin Khoze at Durham University, Lance Dixon discovered that the amplitudes for these processes can be computed by the BCF method by writing the Higgs field as the sum of two complex-valued scalar fields. The amplitudes for producing each scalar are, for the correct choices of gluon polarization, simple expressions of the Parke-Taylor type. For more general choices of gluon polarization, we can break the amplitudes down to the simple case using BCF recursion. Here at SLAC, My Phuong Le and I found that the the search for large extra dimensions of space can be analyzed in a similar way. The signature of extra dimensions is the emission of gravitons from high-energy collider reactions. The graviton is unobserved; what we observe is a recoiling jet and unbalanced momentum. We found that if we assume, just as a fiction, that the graviton decays to two unobserved massless spin one-half particles, we obtain another family of Parke-Taylor amplitudes. In this case also, amplitudes for producing gravitons from gluons of general polarization can be broken down to the simple cases.

A wonderful aspect of the BCF method is that it can be implemented in a computer program without the use of any formula more complicated than the Parke-Taylor formulae. In a Monte Carlo simulation, we randomly generate polarization states for external quarks and gluons. The simulation then sends a request to a subroutine to return the value of the scattering amplitude for those polarizations. The subroutine chops the amplitude into pieces according to the BCF prescription, then calls itself recursively. When the pieces are reduced to the Parke-Taylor form, the expressions are evaluated simply and passed up the hierarchy. The method is not as efficient as the optimal method in the literature, a beautiful but more complex recursion due to Frits Berends (Leiden University) and Walter Giele (Fermilab). But it is very straightforward to program, and it is easily generalized to many interesting LHC processes. Here at SLAC, John Conley, Tommer Wizansky, and I have BCF codes running for a number of LHC reactions, and we are working to incorporate these into useful Monte Carlo programs. We hope that this approach will be helpful for the QCD modelling problems that we will need to solve to extract physics from the LHC data.

—*Michael Peskin,* SLAC Today, *January 3, 2008*