Northeastern University
College of Science

DARPA

"Random Graphs, Simplicial Complexes, and their Applications"

Northeastern University, Boston, MA, May 18-22 2015

Problems of mathematical modeling of large complex systems of varied nature (mechanical, ecological, economical, etc.) motivate studying random geometric, topological, and algebraic objects. For a system of great complexity, it is unrealistic to assume that one may have a precise description of its configuration space, which can instead be viewed as a partially known or random space. Oftentimes, random mathematical objects reflect reality more adequately, while also being simpler to study, since exotic or very intricate examples can be safely ignored.

Random graphs, their limits (e.g., graphons), random simplicial complexes, homology, and their applications in a multitude of fields have seen a surge of interest in recent years. Random graph ensembles are widely used as models of real networks in network science, for example, yet the balance between mathematical rigor and applicability to practical problems remains a major challenge and a continuing interest of funding agencies.

The workshop will bring together leading experts in random graphs and graphons, random geometric graphs, exponential random graphs, random simplicial complexes, and homology theory to overview recent progress in these areas, and to identify future research directions on random discrete structures in mathematics that have a high potential to lead to mathematically rigorous solutions of practically relevant problems.

AGENDA

<u>Monday, May 18</u>

9:15-9:45 Registration

9:45-10:00 Welcome from J. Murray Gibson (Founding Dean, College of Science, NEU)

Welcome from Chris King (Chair, Department of Mathematics)

Welcome from Reza Ghanadan (DARPA), via Skype

10:00-11:00 Michael Farber "Topology of large random simplicial complexes"

11:00-11:30 Coffee Break

11:30-12:30 Richard Kenyon "A variational principle for permutations"

12:30-14:30 Lunch

14:30-15:30 David Rideout "Complex networks as Lorentzian geometries"

15:30-16:00 Coffee Break

16:00-17:00 Ali Jadbabaie "From coverage verification in blind sensor networks to simplicial PageRank"

<u>Tuesday, May 19</u>

10:00-11:00 Matthew Kahle "The threshold for homology with integer coefficients"

11:00-11:30 Coffee Break

11:30-12:30 Yuliy Baryshnikov "Random configurations with topological constraints"

12:30-14:30 Lunch

14:30-15:30 Ginestra Bianconi "Complex quantum network geometries"

15:30-16:00 Coffee Break

16:00-17:00 Jacob Fox "Chromatic number, clique subdivisions and the conjectures of Hajos and Erdős-Fajtlowicz"

Wednesday, May 20

10:00-11:00 Anthony Coolen "New analytical tools for loopy random graphs"

11:00-11:30 Coffee Break

11:30-12:30 Sukhada Fadnavis "Asymptotic properties of the edge-triangle ERGM"

12:30-14:30 Lunch

14:30-15:30 László Miklós Lovász "Left and right convergence of bounded degree graphs"

15:30-16:00 Coffee Break

16:00-17:00 Discussion

18:00 Dinner

Thursday, May 21

10:00-11:00 David Meyer TBD

11:00-11:30 Coffee Break

11:30-12:30 Eric Kolaczyk "On the propagation of low-rate measurement error to subgraph counts in large, sparse networks"

12:30-14:30 Lunch

14:30-15:30 Mei Yin "Asymptotics for sparse exponential random graph models"

15:30-16:00 Coffee Break

16:00-17:00 Yufei Zhao "Large deviations in random graphs"

Friday, May 22

10:00-11:00 Sayan Mukherjee "Stochastic topology: random walks and percolation"

11:00-11:30 Coffee Break

11:30-12:30 Iraj Saniee "Hyperbolicity and centrality in informational networks"

12:30 Adjourn

ABSTRACTS

Yuliy Baryshnikov, UIUC

"Random Configurations with Topological Constraints"

Abstract: I will consider examples of random systems (such as spin models or graph embeddings) subject to topological constraints, and discuss their equilibrium states.

Ginestra Bianconi, Queen Mary University of London

"Complex Quantum Network Geometries"

Abstract: A quantum description of a discrete geometry is a central problem in physics, since it is the necessary to unify Quantum Mechanics with General Relativity. Until now, models for quantum space have focused on the description of discrete homogeneous geometrical spaces. Here, using tools developed in the context of Statistical Mechanics of Complex Networks, we define Complex Network Geometries constructed starting from simplicial complexes and describing the Evolution of Quantum Network States. These networks are geometric networks with energies of the links that grow according to a non-equilibrium dynamics. The evolution in time of the geometric networks is a classical evolution describing a given path of a path integral defining the quantum evolution of quantum network states. The quantum network states are characterized by quantum occupation numbers that can be mapped respectively to the nodes, links, and triangles incident to each link of the network. We call the geometric networks describing the evolution of quantum network states the quantum geometric networks. The quantum geometric networks have many properties common to complex networks including small-world property, high clustering coefficient, high modularity, scale-free degree distribution. Moreover the quantum geometric networks can be distinguished between the Fermi-Dirac Network and the Bose-Einstein Network obeying respectively the Fermi-Dirac and Bose-Einstein statistics. We show that these networks can undergo structural phase transitions where the geometrical properties of the networks change drastically. Finally we comment on the relation between the Fermi-Dirac Network and spin networks.

Anthony Coolen, King's College London

"New Analytical Tools for Loopy Random Graphs"

Abstract: We know that many important graphical structures in the world (biological networks, computing and communication networks, resource grids, lattices in physics, etc) are not tree-like, and that processes on graphs are affected significantly by the presence of short loops. It is therefore problematic that most of our tools and algorithms for analysing (processes on) finitely connected graphs, such as cavity methods, belief propagation type algorithms, and conventional replica analyses, require topologies that are locally tree-like. Some were extended with small loop corrections, but all tend to fail for graphs with many short loops. In this talk I present preliminary results of a new statistical mechanics method that is designed to handle analytically ensembles of large random graphs with prescribed degree sequences and prescribed loop statistics (via their adjacency spectra). It produces explicit closed equations in the infinite size limit, leading to Shannon entropies for ensembles of loopy graphs, and to solutions of models describing processes on such graphs, in the form of free energies and phase diagrams. The familiar equations describing tree-like graphs are recovered as a simple limiting case.

Sukhada Fadnavis, Harvard University

"Asymptotic Properties of the Edge-Triangle ERGM"

Abstract: Diaconis and Chatterjee developed a method for analyzing the limiting behavior of dense

exponential random graphs. They showed that exponential random graph models with non-negative parameters behave like Erdos-Renyi random graph models in the limit. The same is not necessarily true in general. In this talk I will describe the asymptotic properties of the edge-triangle exponential random graph model as the natural parameters diverge along straight lines. This is joint work with Mei Yin and Alessandro Rinaldo.

Michael Farber, Queen Mary University of London
"Topology of Large Random Simplicial Complexes"
Abstract: We study large random simplicial complexes (high-dimensional analogues of random graphs) and their topological and geometric properties. I will focus on a model involving several probability parameters describing the statistical properties of random complexes in various dimensions. The multi-parameter random simplicial complexes interpolate between the Linial-Meshulam random complexes and the clique complexes of random graphs. The Homological Domination Principle states that there is always a Betti number in one specific dimension (the Critical Dimension depending on the probability multi-parameter), which significantly dominates all other Betti numbers. Attempting to understand the general picture of properties of random simplicial complexes with a fixed critical dimension leads to a few conjectures, which I will discuss in my talk. I will also describe some results about probabilistic treatment of the Whitehead conjecture dealing with aspherical 2-dimensional complexes. This is a joint work with A. Costa.

Jacob Fox, Stanford University
"Chromatic Number, Clique Subdivisions and the Conjectures of Hajos and Erdős-Fajtlowicz"
Abstract: A famous conjecture of Hajós from 1961 states that every graph with chromatic number k contains a subdivision of the complete graph on k vertices. This conjecture was disproved by Catlin in 1979. Erdős and Fajtlowicz further showed in 1981 that a random graph on n vertices almost surely is a strong counterexample to the Hajós conjecture. They further conjectured that in a certain sense a random graph is essentially the strongest possible counterexample among graphs on n vertices. We prove the Erdős-Fajtlowicz conjecture. Joint work with Choongbum Lee and Benny Sudakov.

Ali Jadbabaie, University of Pennsylvania
"From Coverage Verification in Blind Sensor Networks to Simplicial PageRank"
Abstract: In this talk I will review some of our previous results obtained through the DARPA SToMP program on how tools from algebraic topology, specifically homology and cohomology theory, can be used to study a range of problems in sensor networks from distributed, location-free coverage verification to intruder detection and coverage repair, as well as distributed estimation based on pairwise relative measurements. Next, I will present some of our recent results on how Hodge Theory and combinatorial Laplacians can be used to define random walks and Dirichlet problems on simplicial complexes, resulting in introduction of a centrality measure analogous to page rank for measuring importance on edges, faces, and higher order cells. Joint work with Alireza Tahbaz-Salehi, Paul Horn, Gabor Lippner.

Matthew Kahle, Ohio State University
"The Threshold for Homology with Integer Coefficients"
Abstract: Linial and Meshulam introduced the topological study of random simplicial complexes, and described the vanishing threshold for homology with field coefficients. In new work, Hoffman,

Paquette, and I show that the vanishing threshold for integer coefficients is the same, at least up to a constant factor. I will discuss these new techniques and mention a few open problems.

Richard Kenyon, Brown University
"A Variational Principle for Permutations"
Abstract: We study limits of measures on permutations of $[n]$ when n tends to infinity. The limiting objects, called permutons, are measures on $[0,1]^2$ with uniform marginals. The size of many naturally occurring classes of permutations can be estimated by maximizing a certain entropy functional over the class of permutons. Unlike the analogous theory for graphs, the limiting objects tend to be analytic and thus often more suitable for classical variational analysis. We compute the permutons for fixed density of inversions and other simple patterns. This is joint work with Daniel Kral, Charles Radin and Peter Winkler.

Eric Kolaczyk, Boston University
"On the Propagation of Low-Rate Measurement Error to Subgraph Counts in Large, Sparse Networks"
Abstract: Our work in this paper is motivated by an elementary but also fundamental and highly practical observation -- that uncertainty in constructing a network graph G^, as an approximation (or estimate) of some true graph G, manifests as errors in the status of (non)edges that must necessarily propagate to any summaries η(G) we seek. Mimicking the common practice of using plug-in estimates η(G^) as proxies for η(G), our goal is to characterize the distribution of the discrepancy D=η(G^)−η(G), in the specific case where η(·) is a subgraph count. In the empirically relevant setting of large, sparse graphs with low-rate measurement errors, we demonstrate under an independent and unbiased error model and for the specific case of counting edges that a Poisson-like regime maintains. Specifically, we show that the appropriate limiting distribution is a Skellam distribution, rather than a normal distribution. Next, because dependent errors typically can be expected when counting subgraphs in practice, either at the level of the edges themselves or due to overlap among subgraphs, we develop a parallel formalism for using the Skellam distribution in such cases. In particular, using Stein's method, we present a series of results leading to the quantification of the accuracy with which the difference of two sums of dependent Bernoulli random variables may be approximated by a Skellam. This formulation is general and likely of some independent interest. We then illustrate the use of these results in our original context of subgraph counts, where we examine (i) the case of counting edges, under a simple dependent error model, and (ii) the case of counting chains of length 2 under an independent error model. We finish with a discussion of various open problems raised by our work.

László Miklós Lovasz, Massachusetts Institute of Technology
"Left and Right Convergence of Bounded Degree Graphs"
Abstract: There are several notions of convergence for sequences of bounded degree graphs. One such notion is left convergence (also known as local or Benjamini-Schramm convergence), which is based on counting neighborhood distributions. Another notion is right convergence, based on counting homomorphisms to a target (weighted) graph. We introduce some of these notions. Borgs, Chayes, Kahn and Lovász showed that a sequence of bounded degree graphs is left convergent if and only if it is right convergent for certain target graphs H with all weights (including loops) close to 1. We will give a short alternative proof of this statement.

David Meyer, UC San Diego
TBD

Sayan Mukherjee, Duke University

"Stochastic Topology: Random Walks and Percolation"

Abstract: The graph Laplacian and random walks on graphs has impacted statistics, computer science, and mathematics. I will motivate why it is of interest to extend these graph based algorithms to simplicial complexes, which capture higher-order relations. I will describe recent efforts to define random walks on simplicial complexes with stationary distributions related to the combinatorial (Hodge) Laplacian. This work will touch on higher-order Cheeger inequalities, an extension of label propagation to edges or higher-order complexes, and a generalization of results for near linear time solutions for linear systems. Given n points down from a point process on a manifold, consider the random set which consists of the union of balls of radius r around the points. As n goes to infinity, r is sent to zero at varying rates. For this stochastic process, I will provide scaling limits and phase transitions on the counts of Betti numbers and critical points.

David Rideout, UC San Diego

"Complex Networks as Lorentzian Geometries"

Abstract: Modeling complex networks in terms of hyperbolic geometry has been enormously successful, for example in providing intuition about a tradeoff between popularity versus similarity in the process of link selection. A recent result describes the hyperbolic model in terms of Lorentzian geometry, and furthermore suggests that the network dynamics is related to gravity. Taking this suggestion seriously, we write down an 'action functional' on complex networks, and take some initial steps at simulating the resulting dynamics by Markov chain Monte Carlo. The results cast new light on a 40 year old problem in enumerative combinatorics.

Iraj Saniee, Bell Labs Research, ALU

"Hyperbolicity and Centrality in Informational Networks"

Abstract: We first provide evidence that informational networks exhibit strong hyperbolicity in the sense of Gromov. We show how hyperbolicity gives rise to maximal centrality for a non-empty set of nodes in such networks. The resulting $O(n^2)$ centrality for a hyperbolic network of n nodes is shown to scale similarly to the centrality due to random walks in arbitrary networks. We leverage the $O(n^2)$ centrality to construct a simple approximation scheme for all-pair distance computations in informational networks that scales like $O(n)$. We provide computational results showing reduction in run times from months/weeks to minutes/seconds thanks to the proposed scheme and the intrinsic hyperbolicity of informational networks. Finally, the application of the proposed scheme to personalized page rank computation and p-center clustering in large-scale information networks is provided.

Mei Yin, University of Denver

"Asymptotics for sparse exponential random graph models"

Abstract: We study the asymptotics for sparse exponential random graph models where the parameters may depend on the number of vertices of the graph. We obtain exact estimates for the mean and variance of the limiting probability distribution and the limiting log partition function of the edge-(single)-star model. They are in sharp contrast to the corresponding asymptotics in dense exponential random graph models. Time permitting, similar analysis will be done for directed sparse exponential random graph models parametrized by edges and multiple outward stars. Part of this talk is based on joint work with Lingjiong Zhu.

Yufei Zhao, Massachusetts Institute of Technology
"Large Deviations in Random Graphs"
Abstract: What is the probability that the number of triangles in an Erdos-Renyi random graph exceeds its mean by a constant factor? The works of Chatterjee--Varadhan (dense setting) and Chatterjee--Dembo (sparse setting) reduce this large deviations problem to a natural variational problem on graph limits. In this talk, I will discuss some results concerning the solutions to these variational problems, which give us large deviation rates for these upper tail events.

PARTICIPANTS

Organizers:
Dima Krioukov, Northeastern University
Chris King, Northeastern University
Alex Suciu, Northeastern University
Gabor Lippner, Northeastern University
Konstantin Zuev, Northeastern University
Reza Ghanadan, DARPA

Invited Speakers
Yuliy Baryshnikov, UIUC
Ginestra Bianconi, Queen Mary University of London
Anthony Coolen, King's College London
Sukhada Fadnavis, Harvard University
Michael Farber, Queen Mary, University of London
Jacob Fox, Stanford University
Ali Jadbabaie, University of Pennsylvania
Matthew Kahle, Ohio State University
Richard Kenyon, Brown University
Eric Kolaczyk, Boston University
Pavel Krapivsky, Boston University
László Miklós Lovasz, Massachusetts Institute of Technology
David Meyer, UC San Diego
Sayan Mukherjee, Duke University
David Rideout, UC San Diego
Iraj Saniee, Bell Labs Research, ALU
Mei Yin, University of Denver
Yufei Zhao, Massachusetts Institute of Technology

Attendees
Rodrigo Aldecoa, Northeastern University
Marco Tulio_Angulo, Northeastern University
Sadjad Asghari-Esfeden, Northeastern University
Michel Buck, Northeastern University
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